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Piezomagnetic Coefficients

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Applying Jahn's method of reduction of a representation and using the authors' method of construction of the magnetic symmetry groups, the significant features of the second order piezomagnetic coefficients for the 90 magnetic crystal classes have been described.

Physical properties of substances generally represent the relation between two quantities each of which may be a scalar, or a vector or a symmetric tensor, *etc.* A physical property is referred to as a magnetic property if one or both of the interacting physical quantities involve the magnetic field, or the magnetic induction or the magnetic moment. Piezomagnetism is the appearance of a magnetic moment on the application of a stress. The possibility of its existence in magnetic crystals has been predicted by Tavger (1958), Dzialoshinskii (1958) and Landau & Lifshitz (1960). Its occurrence has been experimentally verified and measured by Borovik-Romanov (1959) in the fluorides of cobalt and manganese in their anti-ferromagnetic state. The non-vanishing independent first order piezomagnetic constants involving the magnetic moment M and the symmetric polar second rank stress tensor σ have been enumerated by Bhagavantam & Pantulu (1964) employing the character method (Bhagavantam & Suryanarayana, 1949) in respect of the 90 magnetic crystal classes. Sixty-six out of the ninety magnetic crystal classes are shown to be piezomagnetic. The orientations of the magnetic moments of those magnetic structures, which correspond to these 66 piezomagnetic crystal classes, have been described by Koptsik (1966). The number of such magnetic structures has been shown to be 353 when the first order piezomagnetic effects alone are considered.

The first order piezomagnetic coefficients C_{ijk} are studied from the relation

$$M_i = \sum_{j,k} C_{ijk}\sigma_{jk}\ , \quad (i,j,k=1,2,3)\ ,$$

where M and σ stand for the induced magnetic moment and the applied stress. In this note it is proposed to investigate the second order piezomagnetic effects in crystals by including the symmetrized square of the polar second rank stress tensor in the above relation. Thus the appropriate axial tensor is of rank 5 and is magnetic. It is shown here that 69 magnetic crystal classes exhibit second order piezomagnetism.

If V denotes the representation of an axial vector, then the symmetrical product, $[V^2]$ (Tisza, 1933), of V with itself, represents the six components of the symmetric second rank tensor. The appropriate form of the representation pertaining to the second order piezomagnetic effects is represented by $V[[V^2]^2]$. In its reduced form its value is $4D_1+2D_2+3D_3+D_4+D_5$*.

It has already been shown by the authors (Krishnamurty & Gopalakrishnamurty, 1969) that the real one-dimensional irreducible representations of a point group not only induce the magnetic symmetry groups associated with the point group but also give the number of constants required to describe a magnetic property for the induced magnetic symmetry groups. Thus by applying Jahn's (1949) method, one can derive the second order piezomagnetic coefficients for the 90 magnetic symmetry groups, which will be the numerical coefficients of the real one-dimensional irreducible representations of 32 point groups in the reduced form of the appropriate representation. Substituting the reduced form of D_L given by Jahn (1938, 1949) for the point group $\bar{4}3m$ and the axial groups respectively, and by the authors (1968) for the point group $\bar{4}2m$, the second order piezomagnetic coefficients of the 90 crystal classes are enumerated here. They are given below against the distinct inducing real one-dimensional irreducible representations of the 32 point groups in terms of which the 90 magnetic symmetry groups have been derived earlier by the authors (1969):

1:63; $\bar{1}$:63; m:29, $A''(34)$; 2:29, $B(34)$; $2/m$:29, $B_g(34)$; $2mm$:12, $B_1(17)$, $A_2(17)$; 222:12, $B_3(17)$; mmm:12, $B_{1g}(17)$; 4:15, $B(14)$; $\bar{4}$:15, $B(14)$; $4/m$:15, $B_g(14)$; $4mm$:5, $A_2(10)$, $B_1(7)$; $\bar{4}2m$:5, $B_1(7)$, $B_2(7)$, $A_2(10)$; 422:5, $A_2(10)$, $B_1(7)$; $4/mmm$:5, $A_{2g}(10)$, $B_{1g}(7)$; 3:21; $\bar{3}$:21; $3m$:8, $A_2(13)$; 32:8, $A_2(13)$; $\bar{3}m$:8, $A_{2g}(13)$; $\bar{6}$:11, $A''(10)$; 6:11, $B(10)$; $6/m$:11, $B_g(10)$; $\bar{6}m2$:3, $A_2'(8)$, $A_2''(5)$, $A_1''(5)$; $6mm$:3, $A_2(8)$, $B_1(5)$; 622:3, $A_2(8)$, $B_1(5)$; $6/mmm$:3, $A_{2g}(8)$, $B_{1g}(5)$; 23:4; $m3$:4; $\bar{4}3m$:1, $A_2(3)$; 432:1, $A_2(3)$ and $m3m$:1, $A_{2g}(3)$.

Here the figures in brackets appearing against an alternating representation (non-total symmetric real one-dimensional representation) of a crystallographic

* D_L is a representation of dimension $2L+1$ of the group R_∞. Superscripts g or u distinguish D_L according to its being even or odd with respect to inversion. Since piezomagnetism is a centrosymmetric property, the superscript g to D_L is omitted in this note. The notation for the description of the real one-dimensional irreducible representations of the crystallographic point groups is explained by the authors in the preceding paper (Krishnamurty & Gopalakrishnamurty, 1969).

point group give the number of the second order piezomagnetic coefficients required by the magnetic symmetry group induced by that alternating representation of the point group, while the numbers (not in brackets) against a point group indicate the number of the second order coefficients appropriate to the point group. The 21 magnetic symmetry groups, which are induced by the 21 alternating representations of the 32 point groups in each of which the centre of inversion has the character -1, and which do not require piezomagnetic coefficients of any order, are not included in the above list.

It is interesting to note that the number of the piezomagnetic coefficients appearing against the equivalent alternating representations (Krishnamurty & Gopalakrishnamurty, 1969) of a point group will be the same. However, from the equality of the numbers of the piezomagnetic coefficients coming under the alternating representations of a point group one cannot conclude that the representations are equivalent. For instance, from the above list one observes that the alternating representations B_1, A_2 of the point group $2mm$; B_1, B_2 of $\bar{4}2m$ and A_2'', A_1'' of $\bar{6}m2$ are not equivalent.

Further one may also notice that the crystallographic point groups $\bar{4}3m$, 432 and $m3m$ require non-vanishing second order piezomagnetic coefficients whereas no first order constants survive for the three point groups (Bhagavantam & Pantulu, 1964; Koptsik, 1966). The appearance of the second order piezomagnetic coefficients for these three cubic point groups will give rise to a greater number of magnetic structures.

From the very structure of the reduced form of the representation $V[[V^2]^2]$, it is evident that an isotropic solid R_∞^i does not exhibit piezomagnetism since the term D_0 (Jahn, 1949) is absent in the reduced form of the representation.

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Magnetic Symmetry and Limiting Groups

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Using the method of construction of the magnetic symmetry groups already developed by the authors, the magnetic symmetry groups associated with the limiting groups have been derived. The number of constants required to describe the three magnetic properties studied for each one of the derived magnetic symmetry groups is also enumerated.

The crystallographic point groups consist of rotations and rotation-reflexions. Fivefold and higher than sixfold rotation axes are forbidden in the 32 conventional point groups. There is however, a special category of point groups in which infinite-fold rotation axes and reflexions are also permitted symmetry operations. These special groups, seven in number, are called limiting groups (Shubnikov & Belov, 1964), also known as Curie groups. Polycrystalline bodies, fibrous materials like wood, *etc.* belong to a category of substances known as textures. Among a multitude of textures, those possessing the symmetry of limiting groups are of particular interest.

Following the method of the authors (Krishnamurty & Gopalakrishnamurty, 1969) for the construction of the magnetic symmetry groups corresponding to a point group from its real one-dimensional irreducible representations, the magnetic variants of the limiting groups are derived in this note. The numbers of the non-vanishing independent constants in respect of the magnetic properties: (1) pyromagnetism, (2) magnetoelectric polarizability and (3) piezomagnetism, of such